2024

EPSRC Mathematical Sciences Community Workshops 2023

WORKSHOP REPORT

Engineering and Physical Sciences Research Council

Executive Summary

This report details the outcomes of the 2023 EPSRC Mathematical Sciences series of community workshops. The workshops sought to:

- Increase awareness and understanding of how the mathematical sciences community can engage with the EPSRC and UK Research and Innovation (UKRI) delivery plans.
- Explore the strengths and opportunities in the portfolio and consider future strategy plans.
- Work with the community on how we can articulate the value of fundamental science together.
- Support and enhance the relationships between EPSRC and the mathematical sciences community.

The workshops were split into three overarching portfolios:

1. Statistics and Operational Research, held at the International Centre for Mathematical Sciences (ICMS), 17 January 2023.
2. Pure Mathematics, held at the Heilbronn Institute for Mathematical Research (HIMR), 28 March 2023.
3. Applied Mathematics, held at the Isaac Newton Institute (INI), 6 June 2023.

EPSRC would like to acknowledge the support from the three institutes throughout the organisation and hosting of the workshops.

The workshops were designed to capture a snapshot of the current landscape within each overarching portfolio, identifying opportunities and break these down more granularly to understand how these may be realised. Finally, workshops looked to understand how the areas can deliver UKRI research priorities as well as current areas of interest to the community. This is summarised in the figure below:

The workshops highlighted the strength and breadth of mathematical sciences within the UK, and how well placed the mathematical sciences community is to address UKRI, and broader UK strategic priorities. However, it also highlighted the following cross-mathematical sciences challenges:

- Sufficient support for people at all career stages, appropriate to their differing needs.
- Continued support for fundamental mathematical sciences, in particular articulating the benefits of this in a way that resonates with, for example, government.

Elaborating on the challenges and threats to the community, the workshops identified a range of exciting opportunities for the mathematical sciences community. To deliver these opportunities and realise the benefits of mathematical sciences research, the following key recommendations across the breadth of the mathematical sciences remit have been proposed:

- Across the UK there is world-leading research spanning the breadth of the mathematical sciences. It is important for EPSRC to continue to support the fundamental end of these disciplines.
- EPSRC should ensure that their funding and support mechanisms are delivering on meeting the needs of different career stages. Furthermore, EPSRC should improve their signposting of funding opportunities to different career stages to clarify career stage eligibility.
- Communities should strengthen their ability to articulate the value of their work, what makes it interesting, and the outcomes or benefits of the work, as well as articulate how successes in other areas can be attributed to developments from the mathematical sciences. Communities and EPSRC should work together to ensure this is articulated in a way that resonates with stakeholders.
- Communities should better position themselves to respond to other funding opportunities across UKRI, recognising the underpinning nature of the discipline and the role that the breadth of the mathematical sciences disciplines can contribute to realising UKRI's strategic themes.

With respect to the different areas of mathematical sciences, the following recommendations have been proposed:

- Statistics and Operational Research:
    - The current prevalence of Artificial Intelligence (AI) is seen as a strength, weakness and an opportunity to the disciplines. Indeed, whilst the community is well placed to adapt and respond to the current government focus on AI, it must get better at articulating their contributions to the topic and ensuring that Statistics, Applied Probability and Operational Research are at the forefront of discussions.
    - Linked to this, the communities should focus on working out how to articulate their importance to the field whilst not losing their identity as mathematicians.
    - EPSRC should continue to articulate the power of these disciplines within the AI priority, and work to demonstrate how successes within this space can be attributed to mathematical sciences contributions.
    - The discipline is highly connected to a range of different sectors, creating a clear pathway to maximising the impact of the work. The community should consider how to maximise these connections within the wider funding landscape.
    - EPSRC should develop its understanding of the wider UKRI funding picture for Operational Research further to enhance knowledge of the research funded.

  - Given their relevance to the five UKRI strategic priorities, EPSRC should continue to highlight to stakeholders the contribution of the mathematical sciences to realising these ambitions. Furthermore, the community needs to ensure they are able to respond to these funding opportunities when published, maximising opportunities for the discipline to receive funding outside of the mathematical sciences core budget.

- Pure Mathematics:
  - The international reputation of Pure Mathematics within the UK is a strength, and it is important for EPSRC to continue to support blue-sky and curiosity driven research to maintain this international reputation.
  - There are a number of areas of the community working to increase the diversity of those working within the discipline and widen participation, to create a vibrant, diverse academic base throughout the UK, and the community should work on sharing best practice approaches.
  - There is an opportunity for the community to strengthen intra- and cross-disciplinary links, and extend them further to industry and other sectors, and that there is a role for both the INI, the ICMS, HIMR and EPSRC to facilitate this.
  - With limited success in articulating why their proposed topics of interest were exciting, it is evident the community need to be better at articulating this. EPSRC should work to facilitate this in a way that demonstrates the value of investing in fundamental research.

- Applied Mathematics:
  - It is important that EPSRC continues to support applied mathematics across the breadth of the discipline, including both applied and applicable mathematics,
  - The spread of applied mathematics can result in the discipline appearing disconnected, with applied mathematicians not always considered when large multidisciplinary teams are being formed. The community should focus on articulating the benefits of the mathematics to stakeholders to better integrate applied mathematics in programmes.
  - Given the breadth of research that Applied Mathematics underpins, both the community and EPSRC should reflect how to best facilitate connections between different disciplines, recognising the presence of the initiatives delivered by the INI and ICMS, The Knowledge Exchange Hub and network grants that already exist within the landscape.
  - EPSRC should consider whether a discipline hopping scheme would be useful to support knowledge exchange and future proposal development between different areas.
  - Applied Mathematics faces challenges around access to data and managing a cross-Research Council interface.
  - Research support was identified as important to the wider research environment and participants would like to see research facilitator roles at universities and enhanced department-wide support on knowledge exchange, which could be facilitated by the Knowledge Exchange Hub and possibly institutes.

Contents

EPSRC Mathematical Sciences

Research in mathematical sciences is key for the advancement of all areas of science and technology, and a vital area of science in itself. EPSRC aims to sustain core research capability in the mathematical sciences, while promoting transformative and cross-disciplinary research that has the potential for significant impact.

Mathematical Sciences at EPSRC encompasses 12 research areas, which can be approximately grouped into the three higher-level portfolios: Pure Mathematics, Applied Mathematics, and Statistics and Operational Research.

1. Pure Mathematics

The Pure Mathematics portfolio encompasses the following research areas: Algebra, Geometry and Topology, Number Theory and Logic and Combinatorics. Depending on the nature of the mathematics, it may also contain parts of the Mathematical Analysis (research encompassing quantifying change, with a key role played by fundamental notions of continuity and approximation) and Mathematical Physics (research concerned with developing new mathematics inspired by, or relevant to, physics) research areas. The Pure Mathematics portfolio aims to support development of a research and training portfolio that sustains existing excellence and the UK's current position, building on key strengths and nurturing new, leading research topics that aim to further connect Pure Mathematics to other areas within mathematical sciences and beyond. There are strong connections within the Pure Mathematics portfolio, particularly between Algebra and Geometry, although there are fewer connections outside the Pure Mathematics academic base.

The Algebra research area includes mathematical research stemming from study of equations, their solutions and associated operations and symmetries, including group theory, representation theory and ring theory. Algebra is a fundamental, underpinning research area that continues to have significant overlaps with subjects within the mathematical sciences and other research disciplines. Intradisciplinary overlaps continue to emerge, emphasised by significant crossover in research topics with, for instance, combinatorics, geometry, topology, mathematical analysis, mathematical physics and number theory. Links have also been highlighted to other disciplines such as Information and Communication Technologies (ICT).

Geometry and Topology encompasses the study of shape and form, including algebraic geometry, algebraic topology, geometric topology and geometric group theory, differential geometry and geometric analysis. The broad base of research within this field allows it to influence and impact other areas of the mathematical sciences and beyond, for example playing an underpinning role in advances in data science.

The Number Theory research area involves study of the properties of integers, using the tools of modern mathematics to address many basic unanswered questions – for example, concerning the distribution of prime numbers among integers, or solubility in integers of polynomial equations. Number Theory has strong links to all areas of pure mathematics, especially algebra and geometry in relation to the Langlands programme. Other applications include communications technology and, more generally, engineering and physics. Number Theory contributes

significantly to ongoing research in cryptography, an area of national importance as it plays a key role in information security (especially cyber security).

The Logic and Combinatorics research area contains two separate areas of mathematics which have been combined into a single area owing to their relatively small size. Mathematical logic is divided broadly into four areas – model theory, recursion theory (also known as computability theory), proof theory and set theory – that have common origins in the foundations of mathematics, but now have very different perspectives. There is also a strong interface between logic and computer science, including topics such as automated reasoning and program extraction. Links beyond mathematics – for example to computer science and measurement theory – are of national importance due to the role research from logic plays in national security. In its most basic form, combinatorics is concerned with the arrangement of discrete objects according to constraints. Combinatorics studies discrete structures such as graphs (also known as networks) and hypergraphs. This research area includes, for instance, algebraic and probabilistic combinatorics, combinatorial optimisation and Ramsey theory. It has connections to many research areas, for example algebra, mathematical analysis, optimisation, number theory, statistics, theoretical computer science and statistical physics.

Approximately one third to one half of the current EPSRC Mathematical Sciences portfolio is Pure Mathematics. The community receives support from across the EPSRC schemes although are more likely to apply for smaller team or individual funding (including Fellowships and New Investigator Awards) than larger investment mechanisms (such as Programme Grants). In particular, the community submit a high proportion of the postdoctoral fellowship applications received.

2. Applied Mathematics

The Applied Mathematics portfolio encompasses the following research areas: Numerical Analysis, Non-linear Systems, Mathematical Biology, and Continuum Mechanics. Depending on the nature of the mathematics, it may also contain parts of the Mathematical Analysis (research encompassing quantifying change, with a key role played by fundamental notions of continuity and approximation) and Mathematical Physics (research concerned with developing new mathematics inspired by, or relevant to, physics) research areas.

Numerical Analysis encompasses the development, analysis and implementation of algorithms that harness numerical approaches to mathematical problems. It includes research concerned with both computational mathematics (using mathematical methodology to understand discretisation and computation) and scientific computing (designing practical computational algorithms to address challenges in all areas of science and engineering). The ubiquity of numerical algorithms across science, engineering, technology and industry offers considerable potential for numerical analysis to produce substantial and diverse impact through cooperation with other disciplines.

Non-linear systems encompasses research into the mathematical treatment of systems which do not satisfy the principle of superposition (systems where the outputs are not directly proportional to the inputs). These often exhibit richly non-linear behaviour (for example bifurcations,

discontinuities and chaos) and are found throughout engineering, the physical sciences, the life sciences and the economic and social sciences. The area is of considerable importance to a wide variety of other disciplines, application areas and industrial sectors. Non-linear systems research also contributes important theoretical foundations to key research challenges (for example those around data science and urban living). The mathematical foundations of non-linear systems are drawn from dynamical systems, a branch of global analysis overlapping strongly with the mathematical analysis research area. Research in this area may incorporate aspects of complexity science.

Mathematical Biology covers research into the development and application of state-of-the-art mathematical or statistical tools and techniques to investigate biological processes and systems, including those of relevance to the medical sciences. The drive towards increasingly quantitative descriptions of systems and processes in the life sciences has stimulated demand for innovative mathematical and statistical tools and techniques to tackle biological and biomedical challenges. The research area includes research into the development of such tools for the mathematical treatment of biological processes operating at any spatial or temporal scale, or over multiple scales, from the molecular level to the whole population level. It also incorporates novel combinations of existing mathematical techniques or novel applications of mathematics, which inspire new ideas in both the mathematics and biological science areas. Research in this area may incorporate aspects of complexity science.

Continuum mechanics focuses on research into mathematical approaches to the modelling and study of continuous media. Contemporary research in continuum mechanics is erasing the traditional distinction between solid and fluid mechanics and includes mathematical approaches to understanding materials that can exhibit fluid and solid behaviour, as well electromagnetic, biochemical or multiphysical continua. Contemporary developments in this field have led to a broad range of new applications arising from increasing emphasis on developing mathematical descriptions of complex media – for example, photonic crystals, granular materials, dense suspensions, polymers, composites and metamaterials. Understanding such media is of substantial importance in a range of sectors and will provide fundamental insights contributing to high impact research. This research area also addresses the interaction and interfaces between distinct media (for example, in the behaviour of suspensions and granular material, flow in porous media, composites and advanced materials), and may incorporate aspects of complexity science.

Applied Mathematics makes up around 40% of the current EPSRC Mathematical Sciences portfolio, when Mathematical Analysis and Mathematical Physics are included (recognising that these contain pure mathematics as well). Without these overlapping research areas, Applied Mathematics composes around 22% of the mathematics portfolio. In terms of schemes, Applied Mathematics is supported by all EPSRC standard funding schemes, though notably the Mathematical Sciences Small Grants Scheme is very popular. The community has slightly more New Investigator Awards and slightly fewer fellowships compared to mathematical sciences as a whole, though this is more pronounced for postdoctoral level fellowships. Applied Mathematics is highly connected within the portfolio, both to other areas in the mathematical sciences and more widely across EPSRC’s remit. It contributes to high impact research across a broad range of areas, including for societal and economic benefit.

3. Statistics and Operational Research

The Statistics and Operational Research portfolio encompasses the following research areas: Statistics and Applied Probability, and Operational Research.

The EPSRC Statistics and Applied Probability research area covers statistical methodology and the development of new probabilistic techniques inspired by applications, including research in stochastics, probabilistic modelling and inference in stochastics systems. There are strong connections between Statistics and Applied Probability, and an array of applications in sciences, industry, business and government. These provide economic, industrial and societal impact in a range of applications and sectors, including healthcare, finance and the environment. Despite substantial growth since 2015, demand is undiminished for qualified statisticians with an understanding of application areas including data analytics, healthcare modelling and, of increasing importance - artificial intelligence (AI). The research area is the largest within the mathematical sciences, receiving support through all EPSRC schemes available, especially through larger-investment mechanisms such as Programme Grants. The community has higher numbers of New Investigator Award applications but does not tend to apply for post-doctoral level fellowships.

The EPSRC Operational Research research area covers the development and application of advanced analytical methods to support improved decision-making, especially in relation to the operation of complex and uncertain systems. These methods draw heavily on mathematics, statistics and computer science and include, for example: modelling, optimisation, forecasting, simulation, data analysis, stochastics processes and computational research. Operational Research plays a vital role in the UK economy and society, underpinning a wide range of industries and public services, and spans the breadth of UKRI's remit. The research area is the second largest research area within the mathematical sciences, receiving significant support through a number of EPSRC funding opportunities outside of mathematical sciences. The research area also receives significant support from funding calls, for example the manufacturing hubs, trustworthy autonomous systems and through the Industrial Strategy Challenge Fund, reflecting the cross-UKRI nature of the research area.

The Statistics and Operational Research portfolio makes up approximately 20% of the EPSRC Mathematical Sciences portfolio. In terms of schemes, Statistics and Operational Research is supported by all EPSRC standard funding schemes, though notably there are a number of larger investments (Programme Grants and Centres for Doctoral Training). The Statistics and Applied Probability research community submits a higher number of applications to the New Investigator Award scheme compared to the Standard Responsive Mode submissions and in comparison to other parts of the mathematical sciences portfolio, and the portfolio as a whole submit low numbers of applications to EPSRC's fellowship schemes. It is a highly connected portfolio with links across EPSRC's remit, including healthcare, manufacturing, ICT and engineering.

Breakdown of workshop applicants and attendees

Attendance at the workshop was open to any and all researchers working within the mathematical sciences, with colleagues from all backgrounds, a range of career stages, and geographic diversity encouraged to apply. Attendance was determined by an expression of interest process, with 86 expressions of interest submitted. A breakdown of invitees from the expression of interest in terms of self-identified gender, career stage and geographic locations can be seen in Figure 1. From the expression of interest stage, EPSRC invited:

- 19 applicants from 14 institutions to attend the Statistics and Operational Research workshop.
- 19 applicants from 14 institutions to attend the Pure Mathematics workshop.
- 25 applicants from 21 institutions to attend the Applied Mathematics workshop.

Members of the Mathematical Sciences Strategic Advisory Team and Early Career Forum were also invited. Attendee lists for each workshop can be found in Appendix 2.

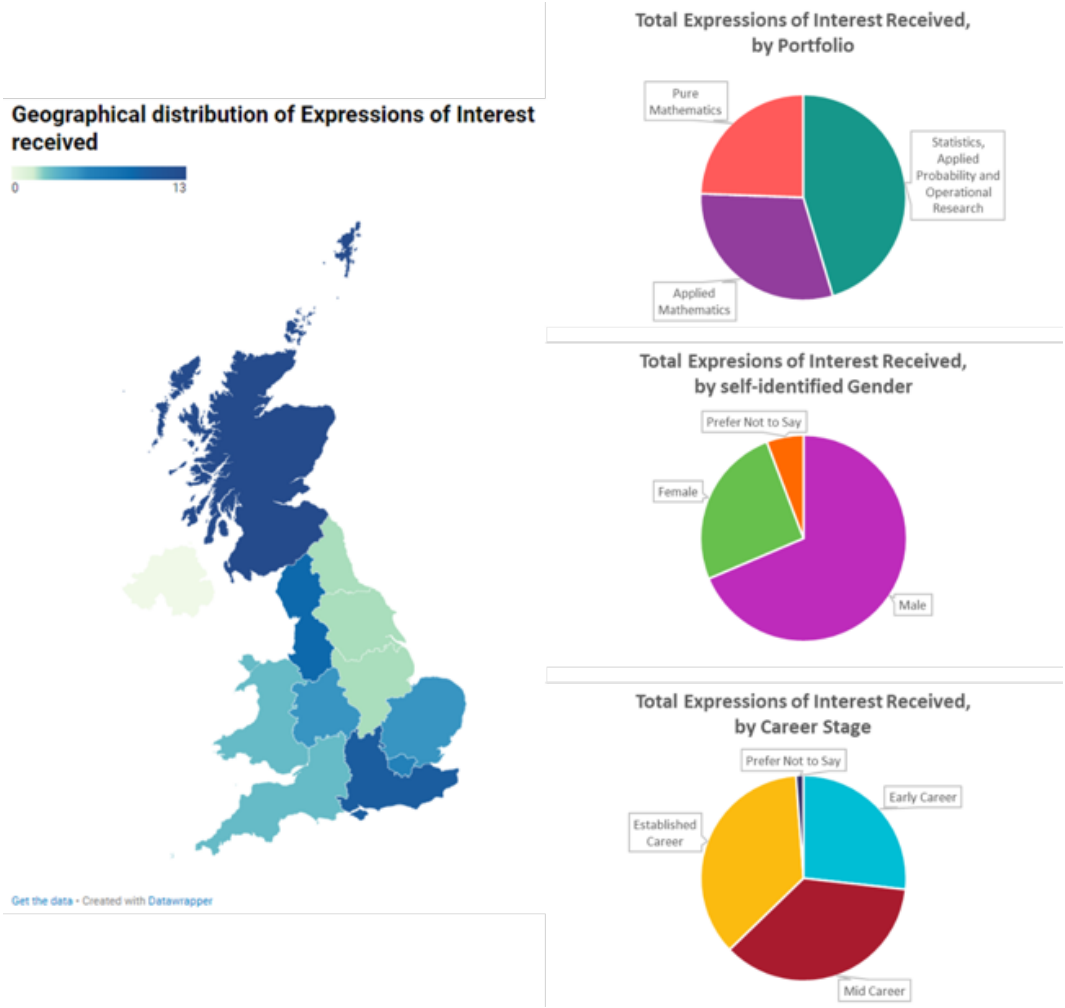

Figure 1: Breakdown of Expressions of Interest received by geographic location, EPSRC portfolio, self-identified gender, and career stage.

Summary of report structure

To address the aims of the community workshop as detailed in the executive summary, each portfolio workshop was broken down into a number of sessions. The agendas for each workshop can be found in Appendix 1.

All workshops started with a session on *Portfolio and Opportunities*, identifying strengths, weaknesses, opportunities and threats to the portfolio, with a summary of the outputs detailed in the *Current State of Mathematical Sciences in the UK* report section.

The following session focussed on further examining the opportunities, identifying barriers to achieving the opportunities and exploring how realising the opportunities would benefit the community. The Statistics and Operational Research workshop also specifically investigated the talent pipeline and support for early career researchers, whilst the Applied Mathematics workshop also looked at the ideal Applied Mathematics landscape. A summary of the outputs can be found in the section *Future Support*.

For the final session, both the Applied Mathematics workshop and the Statistics and Operational Research workshop explored how the disciplines could contribute to the five UKRI strategic priorities. Meanwhile, the Pure Mathematics workshop identified and explored exciting topics in the field with a view to communicating the value of Pure Mathematics. A summary of the outputs from all workshops can be found in the *Future Possibilities* report section.

The report concludes with conclusions drawn as well as any recommendations, both for EPSRC as well as the mathematical sciences community on both a cross-mathematical sciences and portfolio level.

It should be noted in relation to discussions around international collaboration that the workshops were held ahead of announcement of the UK's participation in EU programmes (Horizon Europe and Copernicus).

1. Current State of Mathematical Sciences in the UK

Portfolio strengths

The first session of each workshop looked to identify the strengths, weaknesses, opportunities and threats to the portfolio in the UK. The opportunities identified were further explored, aiming to understand how they could be realised. For this report, we have considered them under UKRI's five strategic objectives or world-class people and careers, places, impacts, innovation and ideas.

World-class people and careers

Across the UK there is world-leading research in the mathematical sciences portfolios, in both theoretical and applied areas. Both the Applied and Pure Mathematics workshops highlighted the strength of departments is supported by a good people pipeline, in that there is not a shortage of candidates for jobs. The Pure Mathematics workshop highlighted career opportunities and funding support for the early career stage as a strength, as well as the ability to attract international talent to the UK, including for permanent positions. High numbers of mathematics undergraduate students in the UK also provides funding to departments. Perhaps connected to this, attendees noted there is strong school outreach and public engagement by mathematics departments in the UK. The diversity of the Statistics and Operational Research community was also noted as a strength, as was the Committee for Women and Diversity in Mathematics.

World-class places

The infrastructure for mathematics in the UK is strong, including high-performance computing excellence and various institutes across the UK, in particular the Mathematical Sciences Institutes (ICMS, INI and HIMR). The Applied Mathematics workshop noted that the programmes and workshops run at the ICMS and INI provide clear benefit to the research community, with the organisation, running and participation in these likened to a research sabbatical. These institutes act as hubs with international reach, connecting the UK to the international mathematical research community. The Pure Mathematics workshop highlighted HIMR as a particular strength for the UK. More generally they noted an increasing shift to centres of excellence which can support high quality research and generate a concentration of talent with an opportunity to drive areas forward; however, this has a negative effect on the geographical diversity of the research community.

World-class impacts

Collaboration was noted as a strength across all three workshops. It was highlighted in the Pure Mathematics workshop that the UK had a welcoming and well-connected community, for example there are a number of existing UK networks and a high number of UK-based conferences and research schools. The community has lots of collaborations both internal to the UK and internationally. Both the Applied Mathematics, and Statistics and Operational Research workshops noted their disciplines were outward facing, active in challenge-solving mathematics and multidisciplinary projects, and well placed to work with external partners (including academic and industry), across different sectors, including healthcare, life sciences, engineering and ICT, with attendees noting that finding academic collaborators is not difficult. Moreover, the disciplines

are able to connect applications to theoretical developments, for example in the development of new models, affording clear pathways to impact in relatively short timeframes. This breadth of areas of impact allows these disciplines to apply for multiple funding streams.

Similarly, Applied Mathematics is flexible and versatile, with a breadth of areas of impact, thanks to its interdisciplinary nature, which connects it to numerous UKRI strategic themes. The grouping of Applied Mathematics together as an academic discipline and within university departments was also identified as a strength in the UK, for example in relation to fluids research. Importantly Applied Mathematics is not just acting as a service department to other disciplines. Attendees noted anecdotally that societal impact from Applied Mathematics in the UK was advanced compared to other countries.

World-class innovation

The Applied Mathematics workshop noted there are hot spots of knowledge exchange development, which is supported by the innovation drive from UK universities, as well as coordination avenues such as the Virtual Forum for Knowledge Exchange in the Mathematical Sciences (V-KEMS), which aims to use mathematics to address challenges from business and industry, the third sector, and other groups outside academia. The connection and dissemination of results provided through the British Applied Mathematics Colloquium, and more broadly through various learned societies, was raised as a further avenue for knowledge exchange.

A spotlight was also shone on the agile and adaptable nature of the community when COVID-19 struck the UK and the Rapid Assistance in Modelling the Pandemic (RAMP) very quickly came into being in March 2020, to address key questions in epidemiological modelling and the behaviour of the virus, notably informing the UK policy response.

World-class ideas

World-leading research in the mathematical sciences was highlighted as a strength across all workshops, with certain topics being flagged as a particular UK strength. The Pure Mathematics workshop identified combinatorics, number theory and geometric group theory, although there is also excellence in other areas. The Applied Mathematics workshop highlighted numerical methods side of quantum technologies as well as Mathematical Biology, with particular strengths including fluid mechanics and continuum mechanics in health, dynamic bioengineering, verification of experimental work, COVID-19 research, and collaboration with Statistics, AI and its applications, along with the numerical methods side of quantum technologies were also mentioned. With the increasing focus on AI, the ability of the Statistics and Applied Probability communities to adapt to topics of interest was also noted as a strength, for example the move towards AI and machine learning from the more traditional Statistics topics, as well as how well students in the area had bought into data science. Indeed, post-graduate training was highlighted as a further UK strength, alongside the ability to influence policy makers, for example in the AI policy agenda.

Both the Pure and Applied Mathematics workshops highlighted that the disciplines do not require huge resources in terms of money and space, beyond some instances of high-performance computing, providing good value for money for investment. A strength noted in the Pure

Mathematics workshop was the small grants scheme landscape in the UK, with offerings from EPSRC, the London Mathematical Society (LMS) and the INI.

Portfolio weaknesses

World-class people and careers

A number of weaknesses were further identified in relation to the people aspect, spanning all career stages.

Insufficient PhD funding was raised at all workshops, with community concerns around the perception that the landscape is dominated by Centres of Doctoral Training (CDTs) to the detriment of other areas of the discipline. The Statistics and Operational Research workshop further flagged the impact of the cost-of-living crisis on the number of students and the potential future impact of it becoming more difficult to hire PDRAs. This was also echoed at the Applied Mathematics workshop, which echoed the difficulties around finding suitable PDRAs as well as a perception of low numbers of postdoctoral-level researchers funded through EPSRC, leaving the community to rely on industry (where the research may have short-term impact) or university fellowships schemes. As a wider issue, the lack of culture around sustaining postdoctoral-level activity through ‘soft money’ (i.e., funding not directly related to grants) is impacting on retention.

Another weakness in the UK is the lack of time for research by staff. A high amount of teaching is required by staff compared to other countries, with time taken up by both teaching and the connected administration. It was noted in the Applied Mathematics workshop that the administration required by academics has increased due to a reduction in support staff at universities. Dedicated time for research in the form of sabbaticals is very difficult or impossible to get at some universities, though by no means at all. The Statistics and Operational Research workshop highlighted this as a particular issue for early career researchers, with a high workload, lack of administration support as well as constrained financial support acutely impacting this career stage.

The Statistics and Operational Research workshop noted the perception that the discipline is a tool, or forms part of a larger project. This means it can be difficult for the community to lead a project, with a large number of community members only being recognised as co-investigators, impacting on career development. Attendees also noted there is a lack of incentives to develop a career in Applied Statistics as well as statistical theory.

More broadly, Brexit was identified as a risk to recruitment and retention across the disciplines.

Despite progress over recent years, diversity remains an important topic with a relative lack of diversity within the Pure and Applied Mathematics communities, for example, in terms of gender and ethnicity, which is seen as a weakness. Some factors that influence this were raised, for example the number of postdoctoral researcher roles expected before securing a permanent job (in particular for the Pure Mathematics community), as well as the requirement to move location in order to meet this.

World-class places

As noted in the Pure Mathematics workshop, concentration of talent gives an opportunity to drive areas forward. However, this has a negative effect on the geographical diversity of the research community.

World-class impacts

The Statistics and Operational Research workshop identified that how their disciplines are perceived can be a weakness. For example, it is difficult for the disciplines to receive recognition in their own right, being used as a tool or forming part of a wider project. Moreover, the value of 'old' models to 'new data' is not always seen as scientific progress by the community, who themselves identified differences in culture across different areas of the disciplines as a further weakness (for example in publication practices).

Whilst the grouping of Applied Mathematics together as a discipline can be viewed as a strength, in practice it can leave it disconnected, with difficulty in reaching researchers who sit outside of mathematical sciences departments (for example, those in Operational Research). As a result of this spread, it is challenging to find out about research others are doing that could help solve your problem. More generally, the community identified that applied mathematicians are not always thought about when large multidisciplinary teams are being put together. There remains a lack of opportunity for some areas in the Applied Mathematics community to engage with others. The Applied Mathematics workshop also identified tension between Applied Mathematics (new applications requiring known mathematics) and the applicable discipline (new Applied Mathematics needing applications).

In terms of multidisciplinary research, the Statistics and Operational Research workshop noted a weakness in the links to social sciences, with a lot of time and resource required to embed into other disciplines. A lack of mechanisms to support and incentivise such multidisciplinary research, especially at an early career stage was also identified as a weakness.

It was also suggested that an area of weakness was in understanding UKRI funding opportunities, to make use of other sources of funding for the community.

World-class innovation

Whilst multi- and interdisciplinary research and working with other disciplines and sectors was noted as a strength it also has associated weaknesses. A lot of time and resource is required to embed mathematics into other disciplines, with the Statistics and Operational Research workshop noting this has been a potential barrier to their engagement with the social sciences. Moreover, Applied Mathematics enjoys strong connections to industry, however, it is held back by misunderstanding remaining around funding, and specifically the business logic of co-funding. Both of these limit the translation of mathematical sciences to addressing real-world challenges.

A specific area of concern in the UK for Applied Mathematics is access to data, more so for mathematical biology research and especially for health. This is due to privacy concerns and the siloed nature of data with large numbers of gatekeepers, reducing the availability of 'real-world' data; Scotland was identified as a place where data is more easily accessed.

World-class ideas

As a result of the fundamental nature of Pure Mathematics it was noted that EPSRC is the majority source to apply for funding, though there is a community perception of low grant success rates. Along with apparent inconsistent funding across panels (attributed to delayed funding decisions stemming from the Additional Funding for Mathematical Sciences programme) this may lead to a lack of engagement of the community with EPSRC.

In the Applied Mathematics workshop, the interdisciplinary nature of proposals is perceived as being disadvantageous in the peer review process. The interdisciplinary nature may mean that there is more opportunity for reviewers to not like or to misunderstand part of the proposal. Moreover, the Statistics and Operational Research workshop raise a concern that interdisciplinary research is judged to lack theoretical novelty, and that given the relatively clear links to other disciplines and sectors, there is a perception that grants must 'tick more boxes' to get funded: the application has to be timely and both the mathematics and the impact must be cutting edge.

From a peer review perspective, both the Applied Mathematics and the Statistics and Operational Research workshop attendees categorised themselves as especially critical or rigorous in their evaluation with one another.

A discussion was held around the current decrease in success rates for Operational Research, in particular in light of the relatively small number of applications received. It was noted that Operational Research spans the breadth of UKRI and currently there is little understanding as to how, and where, Operational Research may be receiving funding support elsewhere in UKRI. Identified as a research-area specific weakness, it was suggested EPSRC should spend some time to better understand the wider funding picture for the discipline within UKRI.

Portfolio threats

World-class people and careers

Career progression was noted as a risk to the UK community at all workshops, with risks identified at all career stages. Student funding levels were identified as a threat, with real term cuts to PhD funding meaning fewer students available through DTPs and CDTs limiting the spread of students. Concerns around the current model for student fees for university income were raised by attendees, however it was recognised this requires change at many levels and is outside the scope of the workshop.

While there are career opportunities and funding support for the early career stage, the Pure Mathematics workshop noted a lack of postdoctoral positions in the UK as well as insufficient numbers of permanent roles at UK universities as a future threat. For the Statistics and Operational Research workshop, attendees noted that research capacity is threatened at the early career stage with teaching and administration burdens a threat to research capacity. Moreover, the required talent to supply the AI sector and other external demands from industry has introduced a threat to the provision of trained researchers in the landscape. This high demand from industry is impacting on recruitment and retention in the academic sector.

The community focus on publications (number, frequency, journal quality), combined with the longer time to publication in some areas compared to others, is also a challenge for the Pure Mathematics disciplines. Increasing hiring competition with the USA has driven the creation of a 'hiring calendar' in the UK and is creating job market fragility.

World-class places

At all three workshops the UK's departure from the European Union and the uncertainty in associating with the Horizon Europe programme was highlighted as a significant threat. The community has lots of collaborations both internal to the UK and internationally; however, it was noted that this may be threatened by external factors such as an increasingly competitive model for UK universities and political factors such as the implications of Brexit and increasing international isolation. The implications identified were that uncertainty around access to grant funding means that UK mathematicians are not being included in funding discussions, leaving the UK community less internationally visible. As a result of the international landscape connected to Brexit and due to competition with other countries, mathematicians are leaving the UK at all career levels, from postdoctoral researchers to established career academics.

Concerns were also raised in relation to the expectation that virtual tools are viewed as a replacement for in-person collaboration impacting of collaboration with international partners.

World-class impacts

Attendees at the Pure Mathematics workshop noted the community may be threatened by an increasing emphasis on real-world and societal impacts of research. It was noted that there is not always a clear or timely link between Pure Mathematics research and impact (for example in the UKRI Strategic Themes). Concerns were raised this may manifest as a loss of funds and an increased need to justify the value of Pure Mathematics research.

Conversely, the Applied Mathematics workshop identified an expectation of immediate impact from research as a threat, with other areas of the discipline less likely to be affected by this expectation. It was suggested that multi- and interdisciplinary grants are disadvantaged in the peer review process, as reviewers from different disciplines often bring counter-perspectives to proposals as well as the reviewers focussing on the impacts of the research rather than the more theoretical contributions.

AI was raised as a potential threat at all three workshops. In addition to being identified as both a strength and an opportunity at the Statistics and Operational Research workshop, AI was also noted as a threat to the identity of the discipline. For example, techniques developed in AI and machine learning, or the role of data-centric engineers may no longer be recognised as mathematics. The focus on AI also poses the threat of detracting from fundamental research, with a need to ensure research in the disciplines in their own right continues (for example, Statistics leading the development and not being used as a tool). This theme was also apparent at the Applied Mathematics workshop, with the fear that AI was losing sight of the mathematics that underpins it, and that it in future it will continue to be based on 'old' mathematics rather than incorporating up-to-date methods. Connected to this is the perception that the mathematics community is reactive to the trends of AI, rather than being proactive and leading the way. Generative AI, with platforms such as ChatGPT, also risk providing a distraction from the bigger

picture. Specifically for Pure Mathematics, while it may be less obvious, there are links to AI. However, a perceived lack of applications for grants in the AI space, including for the Mathematics and Computational Foundations of AI call, was identified as a threat.

World-class innovation

The Applied Mathematics workshop noted the uptake of novel mathematics to improve existing methods is slow, compared to the uptake of solutions.

World-class ideas

Funding was a common theme identified as a threat to UK mathematics across all disciplines. There is a concern that university overheads and rising costs will lead to a reduction in the number of grants that can be awarded.

Mathematics research support from EPSRC is reliant on government funding and there is always a risk it will decrease. There is a perception of a lack of funding being available and that funding has been diverted to areas that the government has identified as a priority. The identification of specific research areas within it also threatens to potentially concentrate research into these areas, leaving out others. Consequently, there is a key need to protect the fundamentals. Concerns were raised as to whether mathematics was leading the development and being integral to the work or being used as a service or tool.

More widely, people outside of the mathematical sciences community do not understand what mathematics research is, which can especially be an issue with government. Applied Mathematics is a UK term, so international reach can be difficult; for example, many applied mathematicians would be considered engineers in the US. This has the potential to be an issue for international reviewers. More generally, a negative perception of the mathematical sciences, or a concern as to whether someone is working in or outside of mathematics means there can be a lack of understanding of the area.

Threats relating to the peer review of Statistics and Operational Research were also flagged: panels not seeing Operational Research as ‘real mathematics’, and interdisciplinary research being judged as having a lack of theoretical novelty by panels are resulting in low success rates, potentially having an impact of the retention of academic staff. Similarly, attendees at the Pure Mathematics workshop noted that research at the interfaces is often perceived as being biased against in the peer review process, even for Pure Mathematics interfaces.

While the interdisciplinary nature of Applied Mathematics is a strength, conversely, they can fall into fall between remits internal to EPSRC and also across Research Councils where they do not obviously fit within the remit of one research council.

Portfolio opportunities

In light of the aforementioned strengths, weaknesses and threats, a number of opportunities were identified.

World-class people and careers

Opportunities to support people in their careers through targeted funding include bolstering collaboration through international funding partnerships to make travel and communication easier, through a joint PI model and in new styles of joint funding, such as collaboration incubation. There are also opportunities for people support, for example in covering non-research costs (for example, visas), introduction of flexible funding, as well as support for early career networks, though no specific funding routes were identified.

In terms of specific schemes, the Applied Mathematics workshop identified investing in postdoctoral fellowships as beneficial, as they are seen as comparatively low cost and have a high impact on the individual and their career progression. There is a perception by workshop attendees that less is spent on postdoctoral fellowships in Applied Mathematics compared to Pure Mathematics and Statistics (including Operational Research). Indeed, EPSRC receive significantly more fellowship applications from the Pure Mathematics community than the Applied Mathematics and Statistics and Operational Research communities.

Small grants are viewed very positively, lowering the barrier to access funding and supporting a greater number of people in the community as they allow more spread of funding and a better diversity of funded projects. The scheme is particularly popular amongst the Applied Mathematics community, though workshop attendees noted continued support of this scheme should not be at the expense of other mechanisms to support postdoctoral researcher positions. Small grants for early career researchers at postdoctoral level were suggested as an opportunity, taking the format of an equitable distribution of funds like the Doctoral Training Partnership (DTP) or Research Associate grants. More generally, doctoral prizes and research associate schemes were identified as popular and an opportunity to continue to provide support at the very early career stage.

Beyond funding mechanisms, the Pure Mathematics workshop identified that mentoring for early career researchers could be implemented to strengthen career opportunities between PhDs and the next role(s) to help in overcoming some of the identified issues.

World-class places

Having the right structures in place is important for bringing people together and enhancing impact. Knowledge exchange platforms, such as the Newton Gateway hub were identified in the Applied Mathematics workshop as allowing community building, information sharing and pump priming activities. The proto-Academy for Mathematical Sciences was noted as an opportunity, with their aim to provide one voice for the mathematics community, including championing the value of mathematics. Developing platforms for sharing problems across the mathematics community was suggested as a useful tool.

Applied Mathematics workshop identified another opportunity connected to place, in ensuring regional diversity of PhD places to attract applicants from across the UK.

The Pure Mathematics workshop noted there is also an opportunity to influence the funding landscape such as, through collaboration with the LMS, looking to fund opportunities in or with other countries, and the introduction of flexible funding, for example not project but person-based or flexible travel grants to promote collaboration.

World-class impacts

Enhanced collaboration in various forms was a common theme for providing opportunities across all three workshops, creating a translational pathway between mathematics research and impact. Multi- and interdisciplinary research was highlighted as needing more support with proposals needing an increased amount of time for preparation to understand the other discipline. A discipline hopping scheme was identified in both the Applied and Pure Mathematics workshops as useful to enable this, particularly for early career researchers, to allow both 'hops' into and out of mathematics. Similar past or present schemes have been available through the Medical Research Council (MRC), the Wellcome Trust and Leverhulme.

The nature of Applied Mathematics lends itself well to connecting to other disciplines, and several areas in particular were identified as opportunities. These include engineering, computer science or ICT as well as bringing together Applied and Pure Mathematics. Especially for the first two areas, enhanced collaborations with other scientists increases the opportunity to access a wider range of funding sources. Connections into other research areas outside of engineering, physical sciences and ICT are less developed, with the humanities and behaviour science identified, which could be brought together in a workshop setting to facilitate connection development.

Pure Mathematics workshop attendees noted this as an opportunity to facilitate links between Pure Mathematics and application areas, as well as interdisciplinary work outside maths more generally. Primarily, the case was made for intradisciplinary research within mathematical sciences. However, it was noted that Pure Mathematics has a role to play in interdisciplinary research in specific application areas such as logistics, computer sciences (in particular in relation to Logic) and the foundations of AI. It was noted however that there can be barriers to this, including mathematical language barriers between different subject areas and thematic snobbery around non-Pure Mathematics research.

Both the Statistics and Operational Research and Applied Mathematics workshop highlighted that the interdisciplinary relevance of their disciplines spanning the breadth of the UKRI portfolio provides a wealth of funding opportunities to which the community can apply. This includes connections to numerous UKRI strategic themes, giving the opportunity to access these additional funding schemes, as well as outside UKRI. The ready path to impact is also an opportunity to raise the profile of the discipline, especially in the Research Excellence Framework (REF) exercise. In terms of discipline-specific opportunities, the Statistics, Applied Probability and Operational Research workshop identified the prevalence of AI, machine learning and data science in the scientific landscape as affording opportunities to maximise impact of research for the community. Participants noted that the community should ride the wave of interest in the area, with opportunities in reliable, explainable and interpretable AI, and in influencing the AI policy agenda. Moreover, integrating data science can lead to new research questions. In a similar way, given the recent public interest in the COVID pandemic, opportunities continue within pandemic modelling and the effective communication and interpretation of data analysis.

World-class innovation

All three workshops identified opportunities connected to strengthening links between mathematics and industry. The relevance of Statistics and Operational Research to industry provides a number of further opportunities, maximising the impact of the work, increasing

knowledge demand, providing a skills demand, employment opportunities and a platform for scientific discussions, and manifesting an additional route for receiving funding. While this is more difficult for the Pure Mathematics community, they could strengthen links between Pure Mathematics and industry or other sectors in areas where there is a link (leveraging for example opportunities provided by the institutes), potentially increasing industry funding in the Pure Mathematics landscape.

Collaboration of applied mathematicians with industry can also be further exploited. The Innovate UK Analysis for Innovators (A4I) programme was suggested as one way that mathematicians can connect with industrial partners, with industry pitching the issue they face in a short video mentioned as working well. More can be done to educate industry on how academia works, for example including industry on funding panels.

In terms of specific topics, the increase in need for high-quality data science (including ethical considerations), underpinned by mathematics, is another important opportunity for Applied Mathematics. Access to health data was highlighted as a particular barrier in mathematical biology, so one opportunity would be an NHS or UKHSA collaboration with UKRI for sharing data at a larger scale to harmonise access and availability. Improving computational infrastructure, including the data infrastructure for industry, would allow digital visibility of phenomena processes for better modelling (for example for supply chains), but partnerships need to be developed to deliver more bespoke solutions.

Other specific subject areas identified as opportunities for Applied Mathematics were industrial mathematical biology and the mathematical modelling of complex systems.

World-class ideas

All three workshops identified that even in a landscape of increasing focus on impact it is key to continue to support blue sky and curiosity-driven research. For example, within Statistics and Operational Research, a balance of support is needed between upcoming topics of interest (for example, uncertainty quantification and digital twins) and fundamental research in these areas.

At the Applied Mathematics workshop, several specific subject areas were identified as opportunities for the community:

- AI, including more balance between mechanistic modelling and AI
- innovative materials (including bio-inspired materials)
- new areas for mathematical exploration, including quantum
- further deepening the strong collaboration between Mathematical Biology and Statistics
- developing the connection between non-linear systems and topological data analysis and topology

Several opportunities at the Applied Mathematics workshop identified by workshop participants were in the mathematical biology and health space. Mathematical biology can contribute to societal benefits, by finding synergistic opportunities. We can become healthier through the use of AI and mathematics in drug discovery, as well as mathematics in health technology. A specific programme was mentioned: African “Global Health” by NHS Scotland. Using mathematics to

examine the health data, providers adjusted the health provision to the local culture, for example how they talked about mental health in different cultures.

Communicating the role and diversity of Applied Mathematics in the UK was a strong theme in this workshop, so the public and policy makers have a greater appreciation of what it does and to attract people into mathematics careers. This can be through outreach in schools or for the general public. Part of this is articulating applied maths approaches more clearly, which can help both in obtaining funding from a greater diversity of sources and to realise impact from their work, tying into UKRI's 'world-class impacts' strategic objective.

Despite being seen as a strength to supporting fundamental research, Small Grants are sometimes less valued by universities over larger grants even when the small grants are ideal for the intended research. Given the low numbers of applications there is an opportunity for the Pure Mathematics community to better engage with the EPSRC Small Grants scheme. Some attendees noted that the time limit of the small grants may be a barrier to applications.

2. Future Support

Realising opportunities

This session aimed to further explore some of the opportunities identified in Session 1, to better understand what actions could be taken to realise the opportunities. Attendees were encouraged to identify how different elements of the landscape can contribute to realising the opportunity, how this might be supported, what barriers may be in place and what successful realisation of the opportunity would look like. Groups were asked to consider this holistically, noting potential differences between different research areas. Note that for the Statistics and Operational Research workshop, attendees selected which to further explore, while for the Applied and Pure Mathematics workshops, these were selected by EPSRC.

Statistics and Operational Research

AI and Data Science

Within the current AI and Data Science landscape there are already a number of driving forces to realise the opportunity. For example, the recent significant investment from government, public interest and domain interest. This is generating new research questions already and resulting in impactful research. However, the rapid growth of the area is resulting in significant competition for funding, and the funding is not geographically well distributed. The rapid growth was flagged by attendees as starting to create staffing problems, particularly in areas where industry has a skills need.

To drive this opportunity forward there needs to be work conducted in the mathematical foundations of AI and data science, defining what is statistical AI or Data Science and where the interface between AI and data science and computer science resides. The community needs to be recognised as a methodological authority, with greater co-creation of questions or solutions to realise real-world impact, including a refocus away from computational power (being driven by the net zero aspiration) as the sole motivating factor.

From a funding perspective there needs to be greater geographic diversity, increased training resources (more teaching of relevant subjects and more consideration of the topics taught), and increased retention of trained people within academia as the shift towards a more data driven society provides opportunities elsewhere.

Recognition that Statistics, Applied Probability and Operational Research are key parts of AI

As identified in the previous opportunity, there is a degree of ‘marketing’ required to ensure that Statistics and Operational Research are integral parts of the AI landscape. Currently, the disciplines have demonstrated that they have the theoretical tools that AI needs and have established links and a good reputation for achieving impact. However, there is a lack of awareness that the disciplines are part of the AI landscape — some in the community don’t always classify themselves as part of the AI community — and there is a perspective that everyone believes they can do Statistics. The communities themselves need to improve at selling their contribution, both to the ‘traditional’ AI communities and emerging stakeholders, potentially through workshops targeting different members of the AI landscape and greater community presence at AI-branded conferences. There needs to be a greater understanding amongst AI experts as to the difficulty and complexity of the Statistics, Applied Probability or Operational Research they are targeting.

From a peer review perspective, there needs to be advertising of opportunities to the most appropriate communities who should be encouraged to apply, and panels and reviewers are required from the relevant communities. Funding is needed to support the retention of PhD graduates, with easy access to individual funds (akin to start-up grants) proposed by attendees as a mechanism to support this though no suitable funding source was proposed by attendees. Moreover, there are different expectations for publications across different stakeholders in the AI landscape which need to be harmonised.

Maximising Impact

Given the clear pathway to realising the impact of work within Statistics and Operational Research, maximising impact was further explored by the workshop attendees. Currently, the community has a clear strength in articulating the impact of their work, as evidenced in the numerous REF submissions. There is also a strong demand from other disciplines, as well as business, industry and governments, particularly in relation to aspects of understanding and training around data science. However, this interest from others can create a barrier in the form of administrative overheads (for example, in putting data agreements in place) and reduces the capacity of individuals in the community. The opportunity to maximise impact is currently further held back by risk appetite from the community, as well as funding opportunities, in particular mechanisms to support multidisciplinary work, and with both mathematical sciences novelty as well as novel application areas.

Looking towards maximising impact, there needs to be a global appreciation of the power of mathematical sciences, and how successes in other areas can be attributed to this. As part of this, there needs to be greater capacity for knowledge transfer beyond the research stage, for example through longer term contacts or having the time to build relationships. From a peer review perspective, there needs to be greater understanding as to whether the process is fit for purpose in relation to impact, recognising impact in the form of both novel mathematics, but also novel application areas.

Strong Early Career Researcher Base

Across Statistics and Operational Research, it is recognised by the community that there is currently strong postgraduate training as well as lectureship opportunities, though the diversity of these could be improved. Moreover, availability of network grant funding provides opportunities for early career researchers to form collaborations and network with colleagues. However, these opportunities could be increased. Additionally, there are currently constraints on the early career base, limiting time to be spent of research activity.

To realise a future strong early career base, the current community-specific networking opportunities need to be extended further, with more students, postdoctoral researchers and early career researchers receiving greater financial support for conference attendance. Attractive salaries already mean that people leave academia for industry, potentially resulting in a talent retention problem for academia. There needs to be greater two-way flow, either of people or knowledge back into academia, as well as support for alternative career paths. Universities should also be encouraged to support this through reviewing their employment terms and conditions, especially in relation to workloads, pay and pensions (though attendees recognised this was an ongoing issue across the sector and beyond the scope of the workshop).

Applied Mathematics

Opportunities in Mathematical Biology

Areas of mathematical biology include epidemiology, healthcare technologies and diagnostics. The benefits of realising this opportunity include ample opportunity for impact, often being 'mission-driven' research and research that meets UKRI strategic objectives relating to an enhanced quality of life. Participants suggested that there is good opportunity for funding in these areas. There is further opportunity for the development and inclusion of machine learning and data-driven approaches.

To realise this opportunity, interdisciplinary research is essential across different academic communities. Sandpit-type events run by EPSRC could facilitate this. There is a need to build ongoing relationships, for which network grants may help, though these may be too focused. One area that mathematical biologists could connect with is control theorists and engineers. It was also suggested that EPSRC could run targeted funding calls in this space.

The Applied Mathematics community does not clearly articulate the benefits to stakeholders, so the community should work to better communicate this. There is inertia from the academic community to engage with one another, which could be tackled by making this as painless as possible. There are also challenges and risks associated with changing discipline, and participants suggested that some 'reward' was needed, perhaps priority for PhD student supervision. Similarly, a lack of opportunities for mid-career researchers was identified, which could be helped by a discipline hopping scheme.

Two specific barriers were identified for the mathematical biology community. First, the availability of data. It was acknowledged that this is hard to solve, though participants suggested that a steer or policy position from UKRI could help in this. There is also the perception that the application of well-known theories to mathematical biology is not considered part of the field. It was noted in discussion that EPSRC had updated the definition of the area to make it clearer that this is considered within remit of the mathematical biology research area, but further promotion by both EPSRC and the community may be needed here.

Data-driven Modelling in Complex Systems

The idea behind data-driven modelling in complex systems is to be clever and effective with the modelling, rather than applying brute force in computational power to solve them. Benefits to realising this opportunity would be the ability to leverage big data and increase the digital visibility of systems. Specifically, the 2013 Deloitte report: Measuring the economics benefits of mathematical science research in the UK identified that there is a large return on investment for being able to model complex systems. Realising this opportunity would increase the interpretability of data, to help improve our understanding by constraining models. One particular area of modelling mentioned is topological data analysis.

Realising this opportunity would require creating the right ecosystem of academia and industry. This would include funding for prosperity partnerships and developing connections between data owners and mathematicians. These connections could be built through a platform for mathematicians and industry to find collaborations, including the knowledge exchange hub and the Knowledge Transfer Network (KTN).

The barriers to realising this opportunity is that there is a disconnect between the modellers and the data owners, and lack of knowledge on who is doing what and where the work would have impact. One suggestion was having panels with more industry experience.

Interactions with Other Disciplines

The benefits of interacting with other disciplines is that it addresses interesting research challenges and can provide motivation for new research questions. It provides access to additional funds and data, and can also enhance impact, while also improving curiosity-driven basic science. For an individual researcher, interacting with other disciplines can provide experience that benefits their career development.

To realise these benefits, the community should exploit existing opportunities, for example they could use network grants more. Interactions with other disciplines also need to reach the community at all levels to respond to opportunities. Participants identified that there needs to be a change in culture in the community to value research at the interfaces as highly as more established areas. Community structures including Knowledge Exchange champions and the INI's Newton Gateway can also be helpful here.

Communication and time were identified as barriers to interacting with other disciplines. There needs to be a common language between them for mathematics to add value, therefore mathematicians should learn the language of the other discipline(s). Similarly, it can take a long time to establish ways of working with the other disciplines. Together these take a long time, which means that researchers are not always motivated to embark on this. There is the risk that the work with the other discipline goes too far downstream to the application area that a lack of a feedback mechanism means that this knowledge is not then shared back with mathematicians for further development. Finally, participants raised that funding cycles restrict when an opportunity in the specific area will arise, with the implication that there is insufficient time from when they are announced to build the understanding and relationships to submit a strong cross-disciplinary proposal.

Articulating the Value of Applied and Applicable Maths

The aim of this opportunity would be a widespread appreciation of the value of Applied Mathematics. In realising this, the hope would be substantially increasing funding from government, which would require the community articulating the value of their research effectively to government and how it aligns to their priorities. Mathematics seeks to solve more general problems, compared to engineering for example, which has one specific and immediate industry problem. This means that solving mathematical problems has a longer 'shelf-life' and can lead to sustained impact, as well as giving a deeper understanding of a problem.

This opportunity could be realised through a communications campaign by the Institute of Mathematics and its Applications (IMA). This would be targeted at government (in particular the Department for Science, Innovation and Technology), industry and academia, with the public as a secondary audience. Another aspect of communication would be in branding, making it obvious that problems within the Applied Mathematics space are in fact mathematics, so more people, including academics in other disciplines, ask to use it. Early communication of the applications of mathematics was also identified as necessary, specifically at the undergraduate level. A system to access mathematics in a more streamlined way was also suggested; this could be a triage

system for mathematical problems to connect and involve industry, academic and government as appropriate. Finally, making more use of network grants could add value in this space.

A significant barrier identified to realising this opportunity was visibility, both in terms of the visibility of Applied Mathematics that is being used for new problems, and the visibility of the versatility of mathematics in tackling them. Finding explicit links between Applied Mathematics and prosperity was another barrier, particularly for government funding. Putting together teams is a challenge, which is not helped by the existence of siloes due to both funding structures and REF, which places impact at the forefront, so the visibility of mathematics can be lost.

Pure Mathematics

Building on the connected nature of the community

Discussion around how to build on the connected nature of the community centred around funding support mechanisms, with no discussion about non-financial aspects in realising this opportunity. The following mechanisms were suggested:

- Keep and encourage Mathematical Sciences Small Grants with the simplified application procedure.
- Network grants can help groups work together, however the workload on maintaining these can be high. Offering dedicated administration support for all networks could mitigate this.
- Existing networks could be funded to grow rather than setting up new networks.
- A ‘Universal Credit’ scheme for researchers. For example, small grants to top up each mathematical sciences department not dependent on a research proposal.

Connections with Industry

There are many areas in which connections with industry may be fruitful. The following areas were identified:

- Machine Learning sector
    - Pure Mathematics providing new problems to hone machine learning techniques
    - Discrete geometry and combinatorics as part of the foundations of machine learning
- Cryptography
- Topological Data Analysis has applications in a variety of sectors and industries
- Coding theory has links to communication technology sector
- Optimisation problems
- Geometric rigidity
- R&D within technology sectors (CSS, Daimler, Bosch)
- Discrete geometry and combinatorics within GPS technology for example, TomTom

There are several key factors impacting the interactions between the Pure Mathematics community and industry. Fundamentally, effective communication between the communities is key to promote this connectivity otherwise neither industry nor the Pure Mathematics community know where the opportunities lie. It was suggested that using the terms pure and applied could

be a barrier to industry involvement. Using current networks to find an ‘intermediate’ researcher (for example, from Applied Mathematics) to facilitate communications could also be used.

There is a perception that within industry there is often a lack of awareness of advanced methods from mathematics, or recognition of the advances that the (pure) mathematics community can offer. It is important to build up a knowledge of valuable propositions for industry, identify compelling case studies within industry parameters, including what can be offered and on what time scales. However, it was noted that the flow of information, resource etc. is very one-way (what can mathematics or mathematicians do for industry?); for successful collaborations there must be benefits to both sides.

There are several barriers to industry working. These include the high risk of research and whether investment will pay off over time. There are limited resources available for initial exploratory work and the effect it has on mathematics research. There is also a lack of domain knowledge within Pure Mathematics which can make it challenging to work with industry.

International Collaboration

As a community, Pure Mathematics is well placed for international collaboration as there is no specialist equipment required. There is a strong role that the institutes can play in international collaboration.

To promote international collaboration several suggestions were made, including short visiting researcher positions for kickstarting collaborative projects, mechanisms to attract and fund high profile international researchers to the UK, international collaboration at the ECR stage for mentorship and profile building, and exploiting opportunities such as funded networks. It was noted that ways to promote collaboration should promote sustainability and creative solutions around collaboration. It was suggested that rethinking international co-investigators on grants to increase flexibility would be of benefit.

There are several barriers to international collaboration that need to be addressed: funding often must benefit the UK – limiting how funds can be spent on international collaborations, and there are governmental barriers to international research (for example in relation to free travel across countries) which restrict the ability of people in certain countries to collaborate with the UK. Linked to this, language barriers can present an additional barrier.

Career Pipeline

The career pipeline looked at three distinct areas of the career progression: PhD to postdoctoral researcher; early career postdoctoral research associate *vs.* fellowships; and postdoctoral to faculty transition.

PhD to postdoctoral researcher — It was questioned whether one-year follow-on positions represent good value for money. It is unclear how the support required may change for top applicants compared to weaker ones. It was also raised whether there is a case for PhD to lectureship transitions within the community, though given the weighting the community places on having a number of postdoctoral positions prior to a permanent position, such an approach would require a significant shift in community expectations around career pathway.

Attendees noted a community perception around postdoctoral researchers *vs.* fellowship positions — in Pure Mathematics (compared to other areas of science) a postdoctoral position is more comparable to that of a fellowship. In spite of this, and whilst there are more fellowships available than in other fields, attendees noted there are still insufficient opportunities. The HIMR fellowship model works well, however these are restricted due to their sensitive nature (by nationality, background etc.) and therefore are not open to all. It was also raised whether the current landscape restricts the ability for new ideas to emerge.

Postdoctoral researcher to senior fellowship or faculty — There needs to be more opportunities for grant applications, such as small amounts of flexible funding (for example, £2-5k per year) for expenses etc. which is flexible over a period of a few years rather than for specific travel or activities. There needs to be a simpler procedure for very small grants compared to the EPSRC Small Grants Scheme; a good model is the Simons Collaborative grants.

Inter- and Intradisciplinary Research

Several barriers to inter- and intradisciplinary research were recorded including limited time and headspace to concentrate on collaboration and translating research, impact on career progression, and a lack of skills in relation to being able to communicate across areas. There is also a community perception that there is a REF penalty where only impact is considered not original mathematical research. It can be a high-risk to move into inter- and intradisciplinary research and it is often unclear if the time and effort spent on this will pay off until an individual has oversight and understanding of both areas. To facilitate this, allowances will have to be made for time taken away from the 'home' area for example, by home organisations in relation to understanding delays in research outputs, and from the community themselves in the assessment of PI track records within peer review processes.

Mechanisms of support for inter- and intradisciplinary research were suggested:

- A funding stream for interdisciplinary thinking (not necessarily a complete project).
- Using the Small Grants Scheme. This could provide time to work out the problem and carry out feasibility activities, however the process should be accepting of failure. Inter- and intradisciplinary research should be flagged as an acceptable use of the scheme, and it is key the panel briefing covers this.
- Providing workshop events to bring together groups to build connections.
- Guidance to panels and reviewers on how to handle inter- and intradisciplinary research.
- Institutes could prioritise inter- and intradisciplinary research.

To promote inter- and intradisciplinary research it is important to collect successful case studies of this type of research and understand how success is measured, as these proposals have higher risk and research is unlikely to be excellent or produce a big breakthrough in both areas. Stakeholders such as university management will need to be influenced to back such research with higher risks.

Having evaluated the current state of the mathematical sciences landscape and developed some of the identified opportunities, two of the workshops discussed areas related to future support. The Statistics Operational Research workshop discussed support for early career researchers, as this had been raised as a community-specific issue. The Applied Mathematics workshop discussed what the future ideal landscape would look like.

Support for Early Career Researchers in Statistics and Operational Research

The aim of this session was to discuss the support for early career researchers and their progression. Attendees explored the reported talent drain from academic to industry, how much of an issue this is and how this affects the community. Further discussion was held around the broader early career funding landscape.

Talent Drain

It is regularly noted during engagement with the community that talent drain from academia, in particular where people move to industry-based positions, is a cause of concern, not least in relation to who will be training the future generation. Academia is perceived as riskier and less financially rewarding than industry, therefore there are fewer factors to keep people in the sector. Moreover, this is making recruitment and retention of post-doctoral level staff increasingly difficult.

It was noted, however, that talent leaving academia to industry is not necessarily negative, and that a balanced two-way flow between industry and academic would be a benefit to the community. It was agreed that the constant evolving nature of the industrial sector also positively shapes the academic research landscape and provides an increased number of funding opportunities and an avenue for more applied research. Moreover, it was noted that industry is perceived as making greater time for research than the academic sector.

Outside of the talent loss from academia, a number of other points were raised in relation to the career pathway and support. There is a concern around mid-career support, with attendees flagging this is only going to become more acute in the coming years given the recent hiring of academic staff at an early career stage. Attendees also noted uncertainty around the future of the discipline, noting that if the current AI and Data Science bubble were to burst, this may have an impact on people. Also linked to this, it was flagged that there is a lack of studentship opportunities in Statistics and Operational Research as the majority of support falls into the remit of AI and Data Science, and student support in the discipline outside of this is limited. Other attendees disagreed, noting that there are overwhelming numbers of students in the space, and flagging concerns about employability.

Factors in relation to the funding processes were raised as being linked to the talent drain. It is becoming increasingly hard to find postdoctoral researchers that will work on grants for three to five years, yet these are critical to the success of the projects. This is more of a concern to this community than for example, Pure Mathematics where the number of postdoctoral researchers supported by grants in substantially lower. New Investigator Awards were flagged by attendees as being less suitable for the Operational Research community, noting that a major motivation for members of this community to conduct a PhD is to move to industry, not academia, therefore there are few people eligible to apply. Whilst Small Grants were positively discussed at the workshop, their relative newness means people are still unsure of how they can use them.

Fellowships are perceived as an intimidating funding scheme, rendering them less accessible as a funding mechanism. Within funding applications, it was suggested that EPSRC should provide more guidance around the training and mentoring expectations for applicants in standard mode grants.

Host institutions were identified as a point at which changes could be made to improve talent retention. Further support for those employed is required, and institutions should exploit their own learned successes to provide this. Salaries were noted as being non-competitive in comparison to industry, and a heavy administration and teaching load, especially at an early career stage, creates an unattractive environment especially compared to the salary, employment conditions, and work-life balance offered by industry. Institutions could also do more to explore the potential around international PhD recruitment.

Early Career Funding Landscape

Discussion focussed on consideration of what the landscape for early career researchers looks like, identifying strengths, concerns, and any gaps in support.

Within the current EPSRC funding landscape, small grants were viewed as a positive addition to the landscape provided that departments uphold any time buyout; in particular it was noted as a positive that they are distinct from The Council of Mathematical Sciences (CMS) small grants scheme. The New Investigator Awards is a good opportunity, especially in relation to providing a base for people to establish their academic careers. However, it was flagged that universities are perhaps putting too much focus on these, rendering them overly important in career trajectory. Consequently, those without one may be perceived as less strong applicants for jobs. Moreover, there was a perception that success in interdisciplinary New Investigator Awards was limited.

Fellowships, both Postdoctoral and Open and Open Plus were also seen as a strength, in particular their flexibility around what can be supported. However, the effort required to put together such an in-depth application does not match up to the heavy administration and teaching load experienced by the early career stage, and low success rates render them as a less attractive funding mechanism. Outside of EPSRC, it was noted that the MRC-staged fellowship process, in which there is feedback at different stages, and the Royal Society Fellowships approach of having a reduced teaching load in the first year could be adopted into EPSRC's fellowship scheme processes to improve support. A discussion was also held around the Research-In-Groups funding from the ICMS and how this can be used.

The previous New Horizons pilot was also noted as a positive funding opportunity for the early career stage. In particular, the anonymous application was perceived as being less biased towards early careers than other funding opportunities. The one year Research Associate positions delivered as part of the Additional Funding Programme for Mathematical Sciences were also raised as a strength, and discussion was had about how this activity would be helpful if repeated.

Within the wider non-UKRI funding landscape attendees identified a number of opportunities for early career funding, highlighting the support offered by the HIMR (travel grants, small pots of

money) as beneficial. Moreover, these have short applications which are more of a match to the time constraints of those at this career stage.

The attendees suggested that there were gaps and improvements that could be made in the current funding landscape. Targeted advertising of schemes, and clearer signposting around the career stage eligibility for different opportunities is a simple way to provide additional support to this career stage. Abridged, or shorter applications processes are more favoured at this career stage reflecting the time pressures experienced. Specific Early Career-targeted calls, for example around visiting scholarships, would be beneficial.

The Ideal Landscape for Applied Mathematics

The aim of this session was to consider what the ideal landscape for the Applied Mathematics portfolio within EPSRC and the UK would look like in 10 years' time. This was broken down into different aspects of the landscape: Funding, People and Careers, Connectivity, Impact, the Wider Research Environment and Other. Attendees were asked to take a holistic view, noting potential differences between different research areas.

Funding

In terms of funding, a number of principles were identified as important to achieve the ideal landscape. Diversity of funding was identified as a priority. Funding should be flexible, robust, fair, quick, reactive and fill a need. It should also realise the Bond Review objectives. Given its underpinning nature, it was suggested that mathematics should be placed at the heart of challenge-led calls. Participants also raised that there should be increased opportunities for mathematicians to be on grants and that UKRI grants should allow more than one Principal Investigator (PI). More radical changes were also suggested, with a radically different funding model with all research active staff eligible for a minimal level of funding, leaving larger proposals much more competitive. In the ideal landscape, there would also be a funding model which includes postdoctoral positions. Another substantial change suggested was for UKRI to provide sabbatical funding.

Student funding was also raised, with requests for individual student funding or students on grants, and small, curiosity-led doctoral level grants that are connected. Attendees suggested in the ideal landscape, there would be a rebalancing of DTPs and CDTs for student funding.

The community are enthusiastic about small grants, which can enable exploratory research. Their ideal landscape would also have seed money or small pots which are agile, including for workshops however how this support would be delivered (for example through EPSRC, learned societies or Institutes), or where such funding would come from was not identified.

People and Careers

For students, the Martingale Foundation scheme was identified as excellent for providing full PhD scholarships to students from low-income backgrounds. In the ideal landscape, participants identified that equivalents for this scheme would be available to cover different aspects of equality, diversity and inclusion, and there would also be postdoctoral research associate diversity initiatives. Postdoctoral training would also be more consistent, and there would be diversity of funding support across career stages and locations. Career movement was another important

consideration, with a more fluid interface between academia, business and government and a well-supported pipeline of career progression forming part of the ideal landscape. The ideal landscape would see research group funding rather than for one PI.

Connectivity

More time was identified as a significant factor in enabling connectivity, with the ideal landscape having dedicated networking time available to academics. Training for postdoctoral research associates on this is also important. As well as time, the right structures need to be in place. An online platform or portal, with a database of all the people in the Applied Mathematics community was identified as useful, which participants proposed could be held by EPSRC or one of the institutes. There is also the possibility to link up with other organisations such as the National Quantum Computing Centre (NQCC).

Funding to enable connectivity could be large consortia grants involving multiple universities tackling interesting problems, akin to the AI hub model. More centre-to-centre support was also identified as helpful for boosting connectivity, as well as creating a discipline hopping scheme. The place agenda is important in considering connectivity, so there should be incentivisation or steers for regional and institutional spread.

Role of Institutes

Public communication is one theme that emerged. When considering what the ideal landscape in 10 years' time would look like, participants identified that institutes would better promote the role of mathematics to industry and other stakeholders. There is a potential that the Academy for Mathematical Sciences would also be up and running. A National Institute for AI (or other subjects), distributed in a geographically diverse way, would also be part of this landscape.

Another common theme was institutes enabling communication within the community, with networking and their role in hosting the knowledge exchange hub being identified as important. Participants indicated they were supportive of the INI, ICMS and HIMR, though they felt there is more scope for Applied Mathematics at them. Finally, participants identified that part of institutes' role is to allocate small pots of money to the community, which could be in partnership with learned societies, for networks and small grants.

The Wider Research Environment

Research support was identified as important to the wider research environment and the ideal landscape would see research facilitator roles at universities and enhanced department-wide support on knowledge exchange, which could be enabled by the Knowledge Exchange Hub and possibly institutes.

Participants would like to see university ownership of diversity (both in terms of equality, diversity and inclusivity factors and in disciplines) and the people pipeline, which would support the UKRI people agenda. In general, some participants identified that there would be less snobbery in the ideal landscape.

Other

Participants identified that the ideal landscape would see reduced bureaucracy, both by funding bodies and universities, for example in the requirement for timesheets.

More general points raised were that the ideal landscape would see more ministers in government who understand mathematics and the value of mathematical sciences research. More awards celebrating achievements in mathematics was also suggested. Finally, participants highlighted that the UK is good at mathematics but needs more money to realise its potential.

3. Future Possibilities

This session aimed to understand future impacts of mathematical sciences research. For the Statistics and Operational Research, and Applied Mathematics workshops, the session aimed to achieve this by understanding how these areas can contribute to delivery of the UKRI strategic themes. For the Pure Mathematics workshop, the session aimed to achieve this by identifying and articulating exciting topics in the community.

Contribution to UKRI Strategic Themes (Statistics, Applied Probability and Operational Research, and Applied Mathematics workshops)

The UKRI strategic themes aim to harness the full power of the UK's research and innovation system to tackle large-scale, complex challenges addressing major national and global challenges. Delegates were placed into groups and encourages to have science-based discussions to develop a 'pitch' as to how Statistics and Operational Research, and Applied Mathematics can contribute to delivering the theme.

- Building a Green Future: This theme aims to accelerate the green economy by supporting research and innovation that unlock solutions essential to achieving net zero in the UK by 2050. Helping improve the health of our environment and deliver net zero, securing prosperity across the whole of the UK. A whole systems approach will secure business growth, jobs, skills and increase productivity, ensuring a green future for all, addressing environmental and net zero challenges in all sectors of the economy.
- Building a Secure and Resilient World: This theme aims to strengthen security and resilience, from individual to national level, across a range of social and economic areas at the heart of daily life. Strengthening social and economic resilience, and enhancing national security across virtual and physical spaces, by improving awareness of risks and threats; preparedness, decision-making and response; and allowing change to be understood as a force for good.
- Creating Opportunities, Improving Outcomes: This theme aims to improve outcomes for people and places across the UK by identifying solutions that promote economic and social prosperity. Understanding the causes and effects of place-based disparities and finding empowering new solutions that promote prosperity and improves outcomes for people and communities across the UK.
- Securing Better Health, Ageing and Wellbeing: This theme aims to improve population health, tackle the health inequalities affecting people and communities, and advance interventions that keep us healthier for longer. Advancing people's health and promoting wellbeing to maintain prosperous, productive and resilient communities throughout the UK and globally, supporting the UK Life Sciences vision by addressing challenges around ageing, living with multiple conditions, mental health and health inequalities.
- Tackling Infections: This theme aims to prepare for future disease epidemics and halt the 'slow motion pandemic' of antimicrobial resistance (AMR). Protecting and enhancing health, our food supply and out natural capital by building knowledge and capabilities to detect and disrupt the emergence and spread of human, animal and plant diseases, and accelerate new vaccines and therapeutics.

1. Building a Green Future

Mathematics can contribute to this theme through building sophisticated models, including real time feedback and control (sometimes called a digital twin). This will help by allowing users to optimise and target the use of new technologies and improve efficiency and safety. The benefit of this will be faster development and deployment of new technologies. Moreover, in the era of big data, machine learning and data science will be useful for interpretation of the data collected.

Applied Mathematics, including fluid mechanics, continuum mechanics and pattern formation techniques can help us understand interconnectivity within the global climate and the effect of changing climate. Dynamical systems, complex systems and multiscale modelling are important for understanding the complexity of the global climate, helping to monitor and identify critical points. These techniques are also important for developing new batteries and fuels, as well as the realisation of tidal energy (identified as underfunded in the UK). The outcome will be better communication of the risks and benefits within climate change and energy resilience to politicians, the public and engineers.

Statistics and Operational research will be important to realising the impact of new and existing technology, for example to optimise the deployment of carbon capture technology, and improving efficiency, for example computational efficiency reducing power consumption, and logistics and transport scheduling resulting in more fuel and time efficient journeys. It will enable accelerated innovation, for example through improved experimental design, new energy efficient algorithms and facilitate the development of novel metamaterials. Robustness, for example propagating uncertainty, model misspecification, and decision-making under uncertainty within the context of energy resilience as well as communication of uncertainty and risk, with interpretable and explainable AI and understanding extreme events will all contribute to being better able to predict and monitor the climate and measure the impacts of new technologies and innovations.

2. Building a secure and resilient world

Both the mathematics of acoustics and solid and fluid dynamics were identified as contributing to a secure and resilient world, contributing to improves earthquake damage protections, flooding and coastal barriers, defence technologies and the development of novel materials. Furthermore, fluid dynamics is important for modelling airflow, which together with behaviour science considering the interaction of people's movement and future outcomes, is important for ventilation, air purification and water supply security.

Mathematical modelling, restructuring behaviour from the real world, using appropriate mathematics to investigate is also important. Key to this modelling is the use of partial differential equations and ordinary differential equations, for example improved modelling of lettuce gravity to realise a reduction in waste in the food chain. Modelling can also contribute to the clear-up of post-chemical weapons attacks and lead to improved protocols for decontamination. Statistics, Applied Probability and Operational Research can further strengthen for mathematical modelling for infrastructural resilience, as well as the development for algorithms for data compression and database strictures for data storage and collation for better data resilience. The disciplines

contributions to ethical, responsible and trustworthy AI, improving awareness of risks, threats and opportunities of the rapidly evolving field.

Network science contributes to energy security by allowing better use of resources through the consideration or redundancy within the network. Other techniques important to this theme include control theory, optimisation using numerical analysis and linear algebra, for example in modelling people's movements in supermarkets.

Statistics and Operational Research were identified as important for accounting for risk within supply chains. Uncertainty quantification and stochasticity underpin data analysis, data interpretation and ultimately decision making under uncertainty and can lead to better decision making for industry, government and an improved quality of life for the public.

It was also noted how Pure Mathematics can also contribute to the realisation of this theme, including the research conducted at the Heilbronn Institute of Mathematical Research in the area of national security, utilising number theory and algebra. Contributions to improved national security are delivered by the Statistics and Operational research disciplines through the development of algorithms for data security, understanding crime and health statistics.

3. Creating opportunities, improving outcomes

Research in this theme often suffers from poor quality data. Techniques here come from a mixture of Applied Mathematics and Statistics, and for this theme it is especially important that the challenges are addressed by interdisciplinary. Mathematics, for example statistical data science, can develop new methodologies for where datasets are noisy or sparse to make predictions, or leveraging imperfect, tiny or complex, messy and incomplete datasets.

Mathematics is also key where models need to be parameterised and for ensuring computation is efficient. It is able to span across multiple difference scales, considering local, national and international levels and is able to consider changes over time, for example through behaviour economics and population-based models. Probabilistic and statistical tools can be developed to understand geographic disparities, for example the inclusion of socioeconomic differences between geographical regions when modelling the impact of funding decision. Mathematics can test models and check feasibility by forecasting solutions, as well as pinpointing other issues. It is able can contribute to unpicking why things happen, for example the evolution of health inequalities. Control theory can be used to operationalise explanatory variables to better understand interventions and provide confidence in decisions, whilst network science, looking to understand how behaviours are influence, can further impact on policy decision. It is also important for policy, enabling modelling-based policy decisions, where statistics, analysis and modelling can be done prior to the implementation of new policies and can further impact on their successful implementation. In addition to development of new models, mathematics can also be used to correctly identify the right model, as well as adapt existing ones to meet changing needs.

Compared to a number of other subjects working in this space, mathematics is relatively cost effective, ethical and may be able to provide greater insight about what is going on, potentially achieving the desired outcome faster. Indeed, the ICMS *Mathematics for Humanity* programme is geared towards the betterment of humanity and another example of mathematics delivering on this theme.

4. Securing better health, ageing and wellbeing

Broadly speaking, many mathematical techniques within mathematical biology will be relevant to this theme, including modelling the economic impact of ageing and mental health, and agent-based modelling including epidemiology. Applied Mathematics can also contribute by modelling inequalities and other factors applied to population dynamics, and through applying deep learning within the drug discovery pipeline helping to advance the field of personalised medicine. More specific techniques include modelling using ordinary differential equations, partial differential equations and topological data analysis.

Statistics and Operational Research can provide overlapping understanding via optimisation, risk analysis, uncertainty quantification and decision-making under uncertainty. Time series models can identify changes in behaviour that could be caused by health problems, and medical statistics, including causal inference, can contribute to improved clinical trials, epidemiology, non-communicable diseases, and survival analysis measures of exposure to harmful environments or substances.

Mathematical developments can further contribute to this theme through increasing economic impact and policy contributions. Other contributions are through robust optimisation, for example for limited resources within the healthcare and social security systems, bed planning and ambulance scheduling. Mathematics can also contribute to mitigating risks, such as disparities in accessing technologies and where technology exacerbates inequalities.

The benefits of these contributions include more robust decision making, including uncertainty handling improved technologies, better scenario simulation prior to implementation.

5. Tackling Infections

Within the tackling infections theme, one important mathematical technique is population dynamics, modelling for example the spread of disease within a population. Multiscale modelling can account for the different scales, including person-to-person (whilst taking into account demographic information), within a building, and within larger environments such as towns. The inclusion of a range of variables forms an in-depth model that can adapt to changes. Mathematics can contribute different types of models, from statistical to stochastic to deterministic. Optimisation can be used to identify the best strategies considering constraints, for example financial or geographical. Other techniques include uncertainty quantification and control theory.

The benefits of involving mathematics in this theme is that it allows consideration of the different scales (from person-to-person to country-wide), demographic information, and uncertainty quantification through optimisation enabling identification of central strategies to achieve the desired outcome. Some examples include changing antibiotic use to minimise resistance development; the introduction of fresh air to the 'hands, face, space' slogan; work on the molecular scale through understanding virus shape. Key to this is that no single person works at all scales, engaging the community is needed to achieve these benefits. Finally, it was suggested: "When you want to change, apply or improve, call an applied mathematician".

Identifying and articulating exciting topics in Pure Mathematics

The session aimed to identify and articulate exciting topics in the Pure Mathematics landscape, noting what outputs and outcomes could be expected from realisation of these topics and why the topics were exciting. A board was left up throughout the morning session for attendees to suggest topics, and a number of these were selected to be further discussed.

1. *Pure Mathematics and AI*

Despite the prevalence of AI in the research and funding landscape, the contribution of Pure Mathematics to the discipline is articulated significantly less than other areas of the mathematical sciences. Indeed, attendees noted that AI is a great example of how the discipline can bloom 30 years later. The following were identified as how Pure Mathematics can contribute to the AI agenda and what outcomes could be realised with its inclusion:

- The mathematics behind AI and machine learning is highly geometrical, for example neural networks. Mathematics can give the precise degree of certainty a neural network will do what you want.
- Exploitation of equivalence and symmetry to improve efficiency within existing approaches.
- Mathematics has the means to demonstrate that the algorithm will work to a high degree of confidence.
- Logic, combinatorics, geometry and analysis are all involved, not just Statistics.
- Mathematics is the justifiable assertions within machine learning.

2. *Applications of Model Theory*

Advances in Model Theory allow for applications in other areas of Pure Mathematics, including complex geometry, group theory and number theory.

3. *Hopf Algebras for Change-Point Detection*

The application of Hopf Algebras for change-point detection is one example of algebraic structures finding unexpected applications and allows anomaly detection.

4. Topology of 4-Manifolds

In comparison to other dimensions, 4-manifolds are of interest as they align with physics (for example relativity, space time, cosmology). Previous work has identified connections between different areas, for example Milnor's exotic spheres led to new connections between homotopy and manifolds, and the topic has the ability to create further connections, linking algebra, geometry, topology, analysis and physics. Moreover, the topic captures mathematical imaginations, for example in revisiting old ideas and solving conjectures (such as solving smooth Poincaré conjecture). Given the UK position in the topic, it attracts world leading researchers to the UK along with excellent students.

5. Proof-Checking

Current work is around the formalisation of as much as possible and needs links to both Computational and Pure Mathematics. It is important as it has the same characteristics as the verification of software without human error, increasing confidence at each stage. Building libraries of mathematical sciences software effectively creates organisation of knowledge. Moreover, it has the potential to impact on the reliability of future mathematical sciences.

It is a hot topic for a number of reasons: more systematic proof (LEAN etc.) with wide applicability, systematic programming within mathematics education, the volume of mathematics and the length of proofs allowing it to spend time at a higher level, and mathematics is more computer trained and more familiar.

The UK was one of the first to work on proof checking with activity now conducted in a few other places (including strengths in France). The topic has the potential to revolutionise how mathematics is verified, with the future vision that when publications are submitted the proof is in a computer programme rather than hundreds of pages of paper, and that proof can be verified by a computer programme.

Conclusions and Recommendations

The workshops highlighted the world-leading mathematical sciences conducted across the breadth of the UK. This scientific excellence ensures that the mathematical sciences community is well-placed to address UKRI, and broader UK strategic priorities. Moreover, the mathematical sciences communities are well connected, either within themselves and to the wider research and innovation community.

However, the workshops also highlighted the following pan-mathematical sciences challenges:

- Sufficient support for people at all career stages, appropriate to their differing needs.
- Continued support for fundamental mathematical sciences, in particular articulating the benefits of this in a way that resonates with for example, government.

All workshops noted concern around the level of PhD student support. While beyond the Mathematical Sciences theme remit, EPSRC appreciate the concerns around rising costs for studentships and the resulting decline in opportunities, noting the three routes for studentship funding: Doctoral Training Partnerships (DTPs); Centres for Doctoral Training (CDTs) and Industrial CASE.

Elaborating on the challenges and threats to the community, the workshops identified a range of exciting opportunities for the mathematical sciences community. To deliver these opportunities and realise the benefits of mathematical sciences research, the following key recommendations across the breadth of the mathematical sciences remit have been proposed:

- Across the UK there is world-leading research spanning the breadth of the mathematical sciences. It is important for EPSRC to continue to support the fundamental end of these disciplines.
- EPSRC should ensure that their funding and support mechanisms are delivering on meeting the needs of different career stages. Furthermore, EPSRC should improve their signposting of funding opportunities to different career stages to clarify career stage eligibility.
- Communities should strengthen their ability to articulate the value of their work, what makes it interesting, and the outcomes or benefits of the work, as well as articulate how successes in other areas can be attributed to developments from the mathematical sciences. Communities and EPSRC should work together to ensure this is articulated in a way that resonates with stakeholders.
- Communities should better position themselves to respond to other funding opportunities across UKRI, recognising the underpinning nature of the discipline and the role that the breadth of the mathematical sciences disciplines can contribute to realising UKRI's strategic themes.

With respect to the different areas of mathematical sciences, the following recommendations have been proposed:

- Statistics and Operational Research:
    - The current prevalence of Artificial Intelligence (AI) is seen as a strength, weakness and an opportunity to the disciplines. Indeed, whilst the community is well placed to adapt and respond to the current government focus on AI, it must get better at articulating their contributions to the topic and ensuring that Statistics, Applied Probability and Operational Research are at the forefront of discussions.
    - Linked to this, the communities should focus on working out how to articulate their importance to the field whilst not losing their identity as mathematicians.
    - EPSRC should continue to articulate the power of these disciplines within the AI priority, and work to demonstrate how successes within this space can be attributed to mathematical sciences contributions.
    - The discipline is highly connected to a range of different sectors, creating a clear pathway to maximising the impact of the work. The community should consider how to maximise these connections within the wider funding landscape.
    - EPSRC should develop its understanding of the wider UKRI funding picture for Operational Research further to enhance knowledge of the funded research area.
    - Given their relevance to the five UKRI strategic priorities, EPSRC should continue to highlight to stakeholders the contribution of the mathematical sciences to realising these ambitions. Furthermore, the community needs to ensure they are able to respond to these funding opportunities when published, maximising opportunities for the discipline to receive funding outside of the mathematical sciences core budget.

- Pure Mathematics:
    - The international reputation of Pure Mathematics within the UK is a strength, and it is important for EPSRC to continue to support blue-sky and curiosity-driven research to maintain this international reputation.
    - There are a number of areas of the community working to increase the diversity of those working within the discipline and widen participation, to create a vibrant, diverse academic base throughout the UK, and the community should work on sharing best practice approaches.
    - There is an opportunity for the community to strengthen intra- and cross-disciplinary links, and extend them further to industry and other sectors, and that there is a role for both the INI, the ICMS, HIMR and EPSRC to facilitate this.
    - With limited success in articulating why their proposed topics of interest were exciting, it is evident the community need to be better at articulating this. EPSRC should work to facilitate this in a way that demonstrates the value of investing in fundamental research.

- Applied Mathematics:
    - It is important that EPSRC continues to support applied mathematics across the breadth of the discipline, including both applied and applicable mathematics,
    - The spread of applied mathematics can result in the discipline appearing disconnected, with applied mathematicians not always considered when large

multidisciplinary teams are being formed. The community should focus on articulating the benefits of the mathematics to stakeholders to better integrate applied mathematics in programmes.

- Given the breadth of research that Applied Mathematics underpins, both the community and EPSRC should reflect how to best facilitate connections between different disciplines, recognising the presence of the initiatives delivered by the INI and ICMS, The Knowledge Exchange Hub and network grants that already exist within the landscape.
- EPSRC should consider whether a discipline hopping scheme would be useful to support knowledge exchange and future proposal development between different areas.
- Applied Mathematics faces challenges around access to data and managing a cross-Research Council interface.
- Research support was identified as important to the wider research environment and participants would like to see research facilitator roles at universities and enhanced department-wide support on knowledge exchange, which could be facilitated by the Knowledge Exchange Hub and possibly institutes.

Appendices

Appendix 1: Workshop agendas

Engineering and Physical Sciences Research Council

MATHEMATICAL SCIENCES THEME

Statistics and Operational Research Community Workshop
International Centre for Mathematical Sciences, Edinburgh, 17th January 2023

17th January 2023		
9.30 – 10.00	**Arrival and coffee**	
10.00 – 10.30	**EPSRC Introduction to Statistics and Applied Probability, and Operational Research**	**Katie and Marianne**
10.30 – 12.45	**Portfolio and Opportunities**	**Katie**
12.45 – 13.45	**Lunch**	
13.45 – 14.30	**Support for Early Career Researchers**	**Marianne**
14.30 – 15.30	**Where is the Maths?**	**Katie**
	(Coffee available during session)	
15.30 – 15.40	**EPSRC Wrap-Up**	**Katie**
15.40	**Close**	

Engineering and Physical Sciences Research Council

MATHEMATICAL SCIENCES THEME

Pure Mathematics Community Workshop
Heilbronn Institute for Mathematical Research, Bristol, 28th March 2023

28th March 2023	
09.30-10.00	**Arrival and coffee**
10.00 – 10.30	EPSRC Introduction to Pure Mathematics
10.30 – 12.00	Pure Mathematics Portfolio
12.00 – 13.00	**Lunch**
13.00 – 14.30	Opportunities
14.30 – 15.00	**Coffee**
15.00 – 16.00	Communicating the value of Pure Mathematics
16.00 – 16.10	EPSRC Wrap-Up
16.10	**Close**

Engineering and Physical Sciences Research Council

MATHEMATICAL SCIENCES THEME

Applied Mathematics Community Workshop
Isaac Newton Institute, Cambridge, 6th June 2023

Tuesday 6 June 2023	
9:30 – 10:00	**Arrival and coffee** Funding landscape proforma available
10:00 – 10:30	EPSRC Introduction to Applied Mathematics
10:30 – 11:40	Applied Mathematics Portfolio
11:40 – 11:50	**Comfort break**
11:50 – 12:45	Realising Opportunities
12:45 – 13:45	**Lunch**
13:45 – 14:40	Ideal Landscape
14:40 – 15:30	Where is the Maths?
	(Coffee available during session)
15:30 – 15:40	EPSRC and SAT Chair Wrap-Up
15:45	**Close**

Appendix 2: Attendee lists for the workshops

Statistics and Operational Research Community Workshop

Professor Selin Ahipasaoglu	University of Southampton
Professor Christine Currie	University of Southampton
Dr Belen Martin-Barragan	University of Edinburgh
Dr Marie Oldfield	London School of Economics and Political Science
Dr Eduard Campillo-Funollet	Lancaster University
Professor Ruth King	University of Edinburgh
Professor Christopher Nemeth	Lancaster University
Dr Anke Wiese	Heriot-Watt University
Professor Peter Challenor	University of Exeter
Professor Jon Forster	University of Warwick
Dr Bin Liu	University of Strathclyde
Dr Ben Swallow	University of St Andrews
Professor Sara Lombardo	Heriot-Watt University
Dr Eric Hall	University of Dundee
Professor Claire Miller	University of Glasgow
Dr Abhishek Pal Majumder	University of Reading
Dr Carolina Euan	Lancaster University
Dr Kaustubh Adhikari	The Open University
Dr Ines Henriques-Cadby	The University of Manchester
Dr Ben Moews	University of Edinburgh
Dr Lisa McFetridge	Queen’s University Belfast
Dr Sebastian Maier	University College London

Applied Mathematics Community Workshop

Professor Chris Breward	University of Oxford
Professor Colm-cille Caulfield	University of Cambridge
Dr Murilo da Silva Baptista	University of Aberdeen
Mr Chris Daniels	FlareBright Ltd.
Dr Andrew Foulkes	Liverpool Hope University
Professor Julia Gog	University of Cambridge
Dr Gabriela Gomes	University of Strathclyde
Dr Chris Guiver	Edinburgh Napier University
Dr Katerina Kaouri	Cardiff University
Professor Sara Lombardo	Heriot-Watt University
Professor Grant Lythe	University of Leeds
Dr Gueorgui Mihaylov	Haleon
Dr Michael Nieves	Keele University
Dr Philip Pearce	University College London
Dr Mariya Ptashnyk	Heriot-Watt University
Dr Hayder Salman	University of East Anglia
Dr Matthew Turner	University of Surrey

Pure Mathematics Community Workshop

Professor Julia Böttcher	London School of Economics and Political Science
Dr Dmitri Finkelshtein	Swansea University
Dr Nicola Gambino	The University of Manchester
Professor Simon Goodwin	University of Birmingham
Dr Johannes Hofscheier	University of Nottingham
Professor Mark Kambites	The University of Manchester
Dr Alexander Kasprzyk	University of Nottingham
Professor Jarosław Kędra	University of Aberdeen
Professor Stéphane Launois	University of Kent
Professor Sara Lombardo	Heriot-Watt University
Professor Jason Lotay	University of Oxford
Dr Daniel Loughran	University of Bath
Professor Nadia Mazza	Lancaster University
Dr John Nicholson	Imperial College London
Dr Anthony Nixon	Lancaster University
Professor John Parker	Durham University
Dr Richard Pinch	Institute of Mathematics and its Applications
Professor Gregory Sankaran	University of Bath
Dr Jason Semeraro	University of Leicester
Professor Ian Short	The Open University
Professor Markus Szymik	University of Sheffield
Dr Lewis Topley	University of Bath